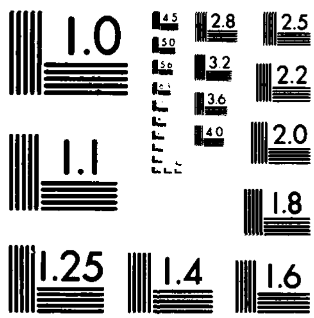
AD-A117 728   BROWN UNIV PROVIDENCE RI LEFSCHETZ CENTER FOR DYNAMI--ETC F/G 12/1
PRACTICAL METHODS FOR THE COMPENSATION AND CONTROL OF MULTIVARI--ETC(U)
APR 82 W A WOLOVICH, P L FALB   AFOSR-77-3182

UNCLASSIFIED   AFOSR-TR-82-0583   NL

END
DATE FILMED
8 82
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 82-0583	2. GOVT ACCESSION NO. AD-A117 728	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) PRACTICAL METHODS FOR THE COMPENSATION AND CONTROL OF MULTIVARIABLE SYSTEMS		5. TYPE OF REPORT & PERIOD COVERED FINAL, 1 JAN 77-31 DEC 81
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) William A. Wolovich and Peter L. Falb		8. CONTRACT OR GRANT NUMBER(s) AFOSR-77-3182
9. PERFORMING ORGANIZATION NAME AND ADDRESS Division of Engineering, Lefschetz Center for Dynamical Systems, Brown University, Providence RI 02912		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS PE61102F; 2304/A4
11. CONTROLLING OFFICE NAME AND ADDRESS Mathematical & Information Sciences Directorate, Air Force Office of Scientific Research, Bolling AFB DC 20332		12. REPORT DATE APR 82
		13. NUMBER OF PAGES 26
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The primary objective of this research has been the development of practical methods for the compensation and control of multivariable systems in order to facilitate the design of controllers for complex Air Force systems. A variety of different approaches were employed to accomplish this objective.

In particular, the authors were most successful in developing new parameter adaptive control schemes for linear multivariable systems. One of the design algorithms employs an adaptive Luenberger observer which (CONTINUED)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ITEM #20, CONTINUED: automatically adjusts the poles of a given system. It is felt that such an adaptive controller could be used in a variety of aerospace applications.

The authors were able to show that "multi-purpose" controllers can be designed which simultaneously perform a variety of control functions. In particular, they constructively demonstrated how to build a controller which simultaneously decouples, places poles arbitrarily, rejects disturbances, insures zero error tracking, and is robust with respect to parameter variations. Additional studies focused on the implementation of such controllers using digital computers. They also presented a new and straightforward method for obtaining simple low order models of systems whose dynamical behavior approximates that of more complex, higher order systems. Such low order models can be used in the design of low order compensators for the more complex systems.

A complete new resolution was also presented to the question of what changes occur to the individual transfer matrix elements of a linear multivariable system under constrained output feedback. In particular, it was shown what poles become controllable and observable via any input/output pair when constraint gain output feedback is applied between any (i-th) output and any (j-th) input. The changes which simultaneously occur to the numerator elements of the transfer matrix were then determined through the employment of some new relationships derived from an approximate relatively right prime factorization of the system transfer matrix.

Another research objective was the study of parameterized system models with a view towards developing compensation and control techniques. The techniques considered involved the methods of algebraic geometry and revolved around three key questions: (i) can the orbits in the space of linear systems under equivalence via the action of an algebraic group be described and classified?, (ii) what spectral structures can be achieved through the use of compensation?, and, (iii) what are the essential elements required in extending results to domains other than the real and complex numbers? Question 1 was resolved for the feedback group and is being studied for systems with parameters. Results have been obtained for the pole-assignment problem involving parameters using intersection theory and some preliminary work has been done on th realization, coprime factorization and trace assignment problems for systems defined over the polynomial ring in n-variables over the integers.

AFOSR-TR- 82-0583

Final

PRACTICAL METHODS FOR THE COMPENSATION AND CONTROL OF MULTIVARIABLE SYSTEMS

AFOSR-77-3182

1. Research Objectives

The primary purpose of the research effort has been and continues to be the development of practical methods for the analysis, compensation and control of multivariable systems such as a high performance aircraft, helicopters, ballistic missiles, and robots. Success in this development facilitates the design of complex Air Force systems. Four critical themes form the foundation for the research; namely: (i) the question of parameter variation which often involves the development of compensators which are insensitive to that variation (i.e. which are robust), (ii) the question of system interaction which leads to a consideration of such things as decoupling, interconnected systems, decentralized control, and qualitative properties; (iii) the question of stability and transient response under compensation which typically leads to a concern with pole-assignment and stabilization; and, (iv) the question of computational effectiveness which generates a search for constructive algorithms with a view towards developing automated design procedures.

On the theoretical level, resolution of these questions has required more and more powerful techniques involving algebra and algebraic geometry. On practical level, the revolution in computer hardware has created an exceptional opportunity for the implementation of complex designs, especially, through the use of microprocessors. Throughout the past effort, the interplay between the theoretical and practical levels is of considerable importance.

Section two of the report contains an historical survey of previous work done by the principal investigators under Air Force sponsorship. The survey will show the natural evolution of the research in the broader context of system science and engineering.

2. Historical Survey

System science and engineering are concerned with the modelling, analysis and control of phenomena both natural and man-made. The discipline, whose formal beginnings go back at least to Watt and Maxwell, derives much of its motivation from control engineering problems. Before World War II, system design and analysis were primarily an art and the key concern was the regulation of simple scalar systems. During and after the war, techniques based on complex variable theory and the Laplace transform were developed and applied to single-input, single-output systems represented by a rational function called the transfer function. The fire control problem was a particularly important application in this era. The theory of servomechanisms developed rapidly from the end of the war through the Fifties and time-domain techniques began to be applied. Most controllers and sensors were analog devices. The beginning of the space age with its guidance and control requirements and the representation of transfer functions via linear constant coefficient differential equations led to an interest in state space methods. The introduction of the Kalman filter and LOG (linear-quadratic-gaussian) design methods had considerable impact on the treatment of multivariable systems. Frequency domain methods using polynomial matrices were also developed. However, the increasing complexity and scale of the engineering and economic problems considered required greater sophistication on both the theoretical and practical levels. Resolution of the theoretical issues has required more and more powerful techniques involving algebra and albegraic geometry. On the practical level, implementation of the more complex design procedures has led to a greater concern with the use of digital computation.

The broad evolution of the field as well as the four crucial themes have impacted the work of the principal investigators. Early work on decoupling and system structure (1), (2) paved the way for considerable later efforts on non-interaction and decentralized control. In addition, results were aimed at providing a practical design approach. To illustrate, the necessary and sufficient condition of (1) led to an easily implemented numerically stable algorithm for pole placement which was used in the manual control of a hovering helicopter (13). Considerable research was devoted to the characterization of those "parts" of a system which remain invariant and those "parts" which can be freely adjusted in various ways. Initial general results occur in (4) and the principal investigators obtained results giving considerable insight regarding the effects of dynamic compensation (5) and feedback(6). In (5), the "interactor" of a transfer matrix was introduced and represents a quantitative as well as qualitative measure of system coupling. This connection was exploited in (7) and has been related to invertibility. In view of the complexity of such design methods as dynamic compensation and in view of the desirability of treating systems with parameter variation, research on adaptive control ((8), (9), for example) and on pole assignment with parameters (10) was undertaken. While initial work on adaptive control involved the model reference approach, a new type of approach was developed which resulted in a combination of on-line identification and control (pole placement). Further, the analysis of adaptive control resulted in an interest in low order models as well as model matching, tracking, and regulation. A new and straightforward method for obtaining simple low order models of systems whose dynamical behavior approximates that of more complex systems was developed (11). It was also shown that "multipurpose" controllers can be designed which simultaneously perform a variety of control functions. In particular, it was shown that a controller could be built which simultaneously decouples, places poles, rejects disburbnces, insures zero error tracking and is robust with respect to parameter variations (12). As the research progressed, it became clear that, on the theoretical level, the powerful techniques of algebraic geometry had a crucial role to play and a broad summary was given in (13). It also became clear that, on the practical level, the most important issue is the question of actual implementation of the final design and that digital computation will play a key role in that implementation.

Summarizing then, significant progress has been made with respect to the development of controllers for multivariable systems, even when parameters vary or are unknown. Numerous techniques are now available for achieving a variety of design goals and, in particular, classical scalar techniques such as root locus and the Nyquist stability criterion have been extended to the multivariable case (14) - (19). Note that (18) and (19) use algebraic geometry in an essential way. The robustness properties of LQG designs have been demonstrated (20), (21) and techniques have been developed for achieving a diversity of design goals simultaneously using a single controller (albeit of rather high order) (12), (22), (23). Adaptive control has also received new impetus in light of recent theoretical developments (24) - (26) coupled with a differential operator approach (27) - (29). However, unlike classical control and compensation techniques which often require little more than pencil and paper computations, many of the modern multivariable procedures require extensive computation. Indeed, computers and computer aided design algorithms are frequently essential in achieving a final design.

3. Applications of Algebraic Methods in Systems Science and Engineering

As noted in the introduction and historical review, there have been four critical themes in the research effort; namely: (i) the question of parameter variation and the devleopment of compensators which are insensitive to that variation (i.e. which are robust); (ii) the question of system structure and interaction which lead to a consideration of such things as interconnected systems, decentralized control, and qualitative properties; (iii) the question of stability and transient response under compensation which results in a consideration of pole assignment; and, (iv) the question of computational effectiveness which involves a search for constructive algorithms with a view towards developing automated design procedures. On the theoretical level, resolution of these questions has involved the use of the techniques of modern algebra and algebraic geometry.

The particular conceptual context of the work revolved around three key questions, namely:

Question 1 - Can the orbits in the space of linear systems under equivalence via the action of an algebraic group be described and classified?

Question 2 - What spectral structures can be achieved through the use of compensation?

Question 3 - What are the essential elements required in extending results to domains other than the real and complex numbers?

A summary of various results and problems is given in (13). In tems of the critical themes, Question 1 relates to the issues of parameter variation, system strucure, and stability; Question 2 relates to these issues and the issue of computational effectiveness; and Question 3 relates particularly to the issue of system structure and more importantly, to the problem of combining digital and continuous systems.

Some of the significant results obtained during the grant period are described and summarized in the sequel.

1. The Interactor and Decoupling

Given any proper rational transfer matrix T(s), a special lower left triangular matrix, $\xi_T(s)$, called the interactor has been defined and shown to be (together with the rank of T(s)) a complete invariant under dynamic compensation (5). Results on decoupling and pole placement were obtained (7). The main theorems are:

Theorem: A system characterized by a nonsingular, proper, rational mxm transfer matrix T(s) can be decoupled via state feedback if and only if the interactor $\xi_T(s)$ is diagonal.

Theorem: Let T(s) be a proper rational pxm transfer matrix of full rank p. Let D(s) be a proper rational pxp diagonal transfer matrix such that $\xi_T(s)\xi_D^{-1}(s)$ is proper. Then there is a dynamic compensator $T_C(s)$ such that $T(s)T_C(s) = D(s)$.

Theorem: A system characterized by a nonsingular, proper, rational mxm transfer matrix T(s) can always be triangularly decoupled with all closed loop poles assigned using state feedback.

These results relate to Question 2.

2. Invariants and Canonical Forms under Feedback

A complete resolution of the problem of determining invariants and canonical forms under feedback for linear systems characterized by proper rational transfer matriices was obtained (6). The invariants were determined in the frequency domain and consist of the Kronecker set of controllability indices together with a canonical form for the numerator of the transfer matrix under the action of a stabilizer subgroup of the unimodular group of polynomial matrices.

Let T(s) be a proper (full rank) pxm transfer matrix so that $T(s) = R_T(s)P_T^{-1}(s)$ where R_T, P_T are relatively right prime polynomial matrices with P_T column proper. The set $\partial_T = \{\partial_1(P_T), \ldots \partial_m(P_T)\}$ is called the Kronecker set of T. If $\partial_1 \geq \partial_2 \geq \ldots \geq \partial_m$, then P_T is properly indexed. Let $n = n_T$ = deg det P_T and let g be an *element of GL(m)*, f be an element of $M_{n,m}$ (= nxm matrices). Call (f,g) a state feedbck pair.

Set

$$P_{T_{f,g}} = g^{-1} \{P_T - fS_{P_T}\}^{\dagger}, \quad R_{T_{fg}} = R_T, \quad T_{f,g} = R_{T_{f,g}} P_{T_{f,g}}$$

Then T_1 is equivalent to T under state feedback if there are state feedback

---

†
$$S_{P_T}(s) = \begin{bmatrix} 1 & 0 & \cdot\cdot & 0 \\ s & 0 & & \\ \vdots & \vdots & & \\ s^{\partial_2 - 1} & 0 & & 0 \\ 0 & 1 & & \\ \vdots & \vdots & & \\ 1 & s^{\partial_2 - 1} & & 0 \\ & \vdots & & s^{\partial_m - 1} \end{bmatrix}$$

pairs (f,g), (f_1,g_1) with $T_{f,g} = T_1$, $T_{1f,g} = T$. This notion defines an equivalence relation. If $O(T)$ is the orbit of T (i.e. equivalence class), then there is a T_1 in $O(T)$ with P_{T_1} properly indexed. Thus, in determining invariants and canonical forms it is enough to deal with properly indexed systems. So, let $W = \{ P \mid P$ properly indexed with Kronecker set $\partial\}$ and let $S_\partial = S(W_\partial) = \{U \mid W_\partial U = W_\partial$, U unimodular$\}$ be the stabilizer of W_∂. Let $X_\partial = \{ R \mid \partial_i(R) \leq {}_i\}$. Then X_∂ is stable under the action of S on the right. The main results are:

<u>Theorem</u> <u>A canonical form for the action of S_∂ on X_∂ exists (and is denoted by R_c).</u>

<u>Theorem</u> <u>A complete system of invariants for equivalence under state feedback is given by (R_c, ∂).</u>

The proof of the first theorem relys heavily on algebraic methods.

3. <u>Pole Assignment with Parameters</u>

Consider the system $x(p) = (A(p), B(p))$ where A,B depend algebraically on the parameter $p = (p_1, \ldots p_r)$. Let $P_n(s) = \{ x(s) : x$ a monic polynomial of degree $n\}$ and let $\Pi_{x(p)}: k^{mn} \to P_n(s)$ be the affine map defined by

$$\Pi_{x(p)}(F) = \det [sI - (A(p) + B(p)F)]$$
$$= \alpha(A(p) + B(p)F \cdot \sigma_n(s)$$

where $\alpha(A(p) + B(p)F) = (1\alpha_1(A(p) + B(p)F)\ldots)$ is the vector of characteristic coefficients and $\sigma_n(s)' = (s^n \; s^{n-1} \ldots s \; 1)$. The problem is to determine conditions which ensure that there is a subspace F_ℓ of k^{mn} such that

$$\Pi_{x(p)}(F_\ell) = P_\ell(s)\psi(s,p)$$

for some $\psi(s,p)$ of degree $n - \ell$ in s. In such a case, it may be said that poles can be assigned independently of p. Results on this problem were derived in (10) for the case $A(p) = A_o + \Sigma\, p_i A_i, B(p) = B_o$. Recently, more general results have been obtained using the methods of algebraic geometry. In particular, there are the following theorems:

Theorem Let $A(p) = A_o + A_1(p) + \ldots + A_\nu(p)$ where A_i is homogeneous of degree i and let $B(p) = B_o$. Suppose that $A(p)$ is non-degenerate with respect to p.* Then n poles can be assigned independently of p if and only if $A_i(p) = 0$ for $i \geq 1$ and (A_o, B_o) is controllable.

(This theorem shows that the problem has content in the sense that if there is true dependence on the parameter, then completely independent pole assignment is not possible.)

Theorem Let $A(p) = A_o + A_1(p) + \ldots + A\ (p)$ where $A_i(p)$ is homogeneous of degree i and let $B(p) = B_o$. Then $\Pi_{x(p)}(F) = V_{x(p)}$ is a subvariety of the affine space of polynomials $P_{n,\nu n}(s,p)$ (monic in s) and so is $\Pi_{x(p)}(F_\ell) = V_{x(p)}(F_\ell)$ for any subspace F_ℓ of k^{mn}. Moreover, there is an F_ℓ such that $V_{x(p)}(F_\ell) = P_\ell(s)\psi(s,p)$ if (i) there is an f_o in F such that $\Pi_{x(p)}(f_o) = s^\ell \psi(s,p)$ and (ii) the varieties $V_{x(p)}(F_\ell)$ and $P_\ell(s)\psi(s,p)$ in $P_{n,\nu n}(s,p)$ are transversal at $\Pi_{x(p)}(f_o)$. (This theorem provides a necessary condition for the problem.)

Calculation of the maximum ℓ for which a solution exists were derived in (10) for the case $A(p) = A_o + \Sigma\, p_i A_i$, $B(p) = B_o$.

4. System Structure

Consider the set of linear system (A,B,C) with A nxn, B nxm and Cpxn. Then GL(n) acts on (A,B,C) via

---

*This means that $R(B_o) \cap \mathrm{Ker}\,(A_i(p)) = (0)\ i > 0$.

$$g \cdot (A,B,C) = (gAg^{-1}, gB, Cg^{-1}),\ g \in GL(n)$$

This action defines state space equivalence and it is natural to ask whether an algebraic quotient exists. In (4), it was shown that the field of invariants $K^{GL(n)} = (k(A,B,C))^{GL(n)}$ is the same as the function field $K(CB, \ldots CA^{2n-1}B)$ of the Hankel matrices. If $S^n_{p,m} = \{ (A,B,C) : (A,B)$ controllable, (A,C) observable $\}$, then the following theorem was proved in (13).

<u>Theorem</u> <u>A geometric quotient $\Sigma^n_{p,m}$ for state space equivalence exists. Moreover, $_{p,m}$ is a rational, smooth, irreducible quasi-projective variety of dimension $n(m+p)$.</u>

This theorem has applications to system identification and to the determination of generic properties.

Increasing interest in image procesing and in the interconnection of digital and continuous systems and in systems with fixed delays has led to a consideration of systems involving transfer matrices whose entries are rational functions in n-variables. A key result that was obtained is the following lemma:

<u>Lemma</u> Let R be a Noetherian integral domain of characteristic 0 and let Z be the integers. Then the realization and coprime factorization problems are solvable over $R[x_1,\ldots,x_n] = Z[x_1,\ldots,x_n] \otimes R$ if and only if they are solvable over $Z[x_1,\ldots,x_n]$ and R is a regular extension of Z.

This lemma exposes the essential algebraic nature of certain basic system problems and has considerable implications for system structure.

5. Adaptive Control and Model Reduction

The main thrust of our work in this area has been the development of practical algorithms which can be used to adaptively control multivariable systems. At the outset of this work, considerable progress had been made in designing globally stable adaptive controllers for unknown continuous time scalar systems (24) - (26). Little progress had been made in extending such designs to the multivariable case. This was remedied, in part, by the multivariable adaptive control scheme detailed in (28). In particular, in (28) an equation is derived that can be used to identify a set of controller parameters which completely assign all poles and zeros of a linear multivariable systems represented by a (pxm) strictly proper transfer matrix T(s) when $p \geq m$ and only input-output data is available for measurement. This equation is used to derive a parameter adaptive control scheme for linear multivariable systems.

The control structure contains an adaptive Luenberger observer which assigns as poles of the closed loop system the zeros of the unknown system (plant), and possibly some additional poles specified by the designer. The new overall system zeros and the remaining poles are obtained by use of a fixed precompensator which can be arbitrarily specified by the designer. Since the incorporated parameter identifier directly estimates the control parameters without explicitly identifying a parameterized model of the unknown plant, the structure can be classified as a direct adaptive controller.

Since the plant zeros are assigned as closed loop poles, to construct the controller one must be assured that the plant's open loop zeros lie strictly in the left half plane. In addition, to complete the design one must know, a priori, an upperbound on the plant's observability index, as well as know the structure of the interactor matrix (5) associated with T(s). In many cases, this latter requirement is essentially equivalent to knowledge of the smallest relative degree in each row of T(s). To assure proper performance additional information about the plant transfer matrix structure is also needed. This information is analogous to information on high frequency gain necessary for adaptive control of scalar systems (24). It might be noted that the design presented in (28) is more general than those presented in earlier reports since it is applicable to a larger class of multivariable systems.

More recently, the restrictions associated with this direct adaptive controller have been partially overcome by the development of a new adaptive control structure for MIMO systems. It is applicable to the class of systems characterized by proper full rank transfer matrices, and can be implemented with only a priori knowledge of the system controllability indices. The adaptive scheme is based upon a fixed observer structure, and arbitrarily assigns the closed loop system poles. The algorithm can be viewed as a generalization of the SISO scheme for direct adaptive pole placement discussed in (30). A preliminary version of this technique was reported at the Twentieth Decision and Control Control Conference (31). However, that report did not allow for completely arbitrary pole assignment, and the modelling assumptions considerably restricted the class of systems for which it was applicable. The results given in (32) overcome these problems.

The problem of finding reduced order models for high order systems, sometimes referred to as the "model reduction problem", is an important one to the practicing engineer since it is difficult to apply the design procedures of modern and classical control theory to high order systems. Numerous solutions have been proposed during the past two decades and the survey of Bosley and Lees (33) represents an excellent summary of the available methods. A number of methods are based on first deriving transfer functions or state-space models for the high order systems and then simplifying these models, e.g. (34), (35), (36). Other methods use time or frequency response data to directly fit low order models (37), (38), (39). Our procedure falls into this latter category, since it determines a model which matches the frequency response of the original high order system at a certain set of prespecified frequencies. Its primary advantage lies in the simplicity of implementation. In particular, no intermediary high order model need be calculated, only one test input need be used, and the calculation of model parameters only requires the solution of a simple set of linear equations. The model parameters can also be obtained as the output of an analog adaptive network, since the algorithm makes use of the generalized equation error identification scheme proposed by Lion (40). Most importantly, our algorithm readily generalizes to the multiple input-multiple output (multivariable) case where classical frequency and time domain procedures become cumbersome to apply.

More specifically, in (11), we present a new method for obtaining simple low order models whose dynamical behavior approximates that of more complex, higher order, stable linear systems. The low order model is determined by applying an identification procedure to input-output data obtained by driving the original system with a special periodic test signal. We prove that in the scalar case a Lion-type model adjustment identifier will determine a constant k^{th} order model of an n^{th} order ($k < n$) system provided the system input consists of exactly k distinct sinusoids. This k^{th} order model will approximate the higher order systems in the sense that its frequency response matches that of the model at the k input frequencies (provided the model obtained is stable). We then show that this result can be extended in a very natural way to the multivariable case. We finally demonstrate by example that this procedure can produce excellent low order models when such models exist.

6. Multi-Purpose Controllers

Frequency domain mothods have always dominated control system design in the scalar (single input/output) case, when compared to the more "modern" state-space or differential operator methods, due to the relative simplicity of the resulting controllers and their ability to function acceptably over a rather wide range of plant parameter variations; i.e. their robustness. It is not surprising, therefore, that numerous studies have been made to "extend" various frequency domain techniques to the multivariable case in order to simply and reliabley achieve a diversity of desired design goals. In most cases, however, direct extensions of scalar frequency domain procedures, such as the Nyquist stability criteria or the root locus, are not possible and often rather complex modifications have to be made to existing theories in order to achieve appropriate design objectives. Further complicating the picture is the fact that noninteraction (or decoupling) is often an additional design objective in the multivariable case, and a completely decoupled, stable system cannot always be achieved by the relatively simple feedforward controllers obtained by multivariable, frequency domain methods.

On the other hand, the so called "modern" methods which have generally relied on exact knowledge of the plant, are continually being improved upon and extended to take into consideration parameter uncertainty and/or variations; i.e. robustness is becoming increasingly important in designs based on state-space or differential operator methods. Although these "modern" methods generally imply more complex controller configurations, than those associated with frequency domain methods, they are less heuristic to implement and can generally achieve more than is possible with the simpler controllers designed by frequency domain methods. Moreover, with the ever increasing utilization of computers in the control loop, it may be argued that controller simplicity is no longer as important as it once was, and one might therefore expect to see more complex controllers being used in future applications.

In light of these observations, a new procedure has been developed for designing controllers which simultaneously achieve a variety of desired design goals in deterministic, unity feedback, linear multivariable system. This procedure involves an appropriate combination of work delineated in three earlier reports, the first dealing with the pole placement stabilization of stabilizable systems (41), the second with skew prime polynomial matrices and their importance in extending the internal model principle to the multivariable case (42), and the third with output regulation and tracking with zero steady-state error (43). All three of these efforts are appropriately combined in (12) to yield a new algorithm for the systematic design of a "three part" multivariable controller which simultaneously insures

(a) a non interactive or decoupled closed loop design, well as closed loop stability,

(b) complete and arbitrary closed loop pole placement, which implies desired (single loop) transient performance as well as closed loop stability,

(c) zero steady-state errors between the plant outputs and any nondecreasing deterministic inputs,

(d) complete steady-state output rejection of nondecreasing deterministic disturbances, and

(e) robustness with respect to stability, disturbance rejection, and zero error tracking for rather substantial plant parameter variations.

Our development employs the more "modern" (Laplace transformed) differential operator approach (27) for controller synthesis, which involves transfer matrix factorations and the manipulation of polynomial matrices in the Laplace operator s.

Reference (43) (Remark 6 in particular) indicates how one might translate, to differential operator form, the results in (44), (45) and (46) in order to design a robust compensator which yields <u>either</u> regulation or tracking. These results are extended in (12) to include simultaneous regulation and tracking using a single robust "internal model", M^{-1}, of both the disturbance and the reference. We also show how the remainder of our feedforward compensator can be chosen, along with the decoupled open loop poles, to achieve an overall set of desired closed loop decoupled poles under unity feedback. Most importantly, we have successfully "combined" the results given in the earlier references to present, for the first time, a formal proof of our ability to achieve all of the desired design goals simultaneously.

7. Constrained Control

In the control of linear multivariable systems, constraints are often imposed on both the complexity of the controller as well as its placement relative to the system's inputs and outputs. An interesting question relative to this observation is what changes occur to the dynamical relations between any input/output pair as the result of applying constant gain output feedback between any (i-th) output, y_i and any (j-th) input, u_j. This question is clearly related to that of decentralized control; i.e. of determining the conditions under which one or more "local" output feedbacks can be applied to insure complete state controllability of the system through a selected input or set of inputs. The solution to the rather general question which is posed in (47) contains the elements for resolving the decentralized control question, as well as other "constrained" control questions, in a new and efficient manner.

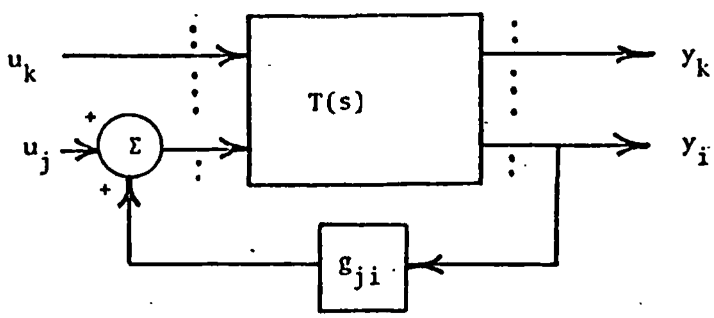
The particular treatment employed in (47) assumes knowledge of the (pxm) rational transfer matrix, T(s), which characterizes the dynamical behavior of the system under investigation. By then employing a relatively right prime factorization, $R(s)P(s)^{-1}$, of T(s) with P(s) in unique (upper right triangular) Hermite form, significant new insight is obtained relative to the changes which occur in all of the pxm elements of T(s) when any output y_i is fed back to any input, u_j through a constant gain element g_{ji}.

To be more specific, let us consider a linear multivariable system whose dynamical behavior is specified by a (pxm) proper, rational transfer matrix $T(s) = [t_{ij}(s)]$. Let us further assume that n denotes the order of any minimal (state-space) realization of T(s), and that all n poles of T(s) are distinct (such an assumption serves to simplify this presentation, but is not really necessary in general). If each rational element $t_{ij}(s) = \frac{r_{ij}(s)}{p_{ij}(s)}$, of T(s) is then represented in prime, factored form, one can immediately determine th "controllability/observability properties" of each of the n modes or poles by inspection of T(s). To illustrate, if $p = m = 3$, and

$$T(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{s+1}{s+4} & \frac{1}{s+1} \\ \frac{1}{s+2} & \frac{1}{s+4} & 0 \\ \frac{1}{s+3} & \frac{s+2}{(s+3)\ (s+5)} & \frac{s+1}{s+6} \end{bmatrix}$$

then it is clear the T(s) has 6(=n) distinct modes at s = -1, -2, -3, -4, -5, and -6. Furthermore, since the mode at s = -1 appears only in the first row and first and third columns of T(s), it is observable only at the first output, y_1 and controllable only by the first and third inputs, u_1 and u_3, respectively. Similar observations can readily be made regarding the other (five) system modes; e.g. the mode at s = -4 is controllable only by u_2 and observable only at y_1 and y_2.

One of the fundamental questions in constrained control is the following: What system modes become controllable via u_k if y_i is fed back to u_j via an arbitrary gain g_{ji}, as depicted in the figure below? It might be noted that only the outputs and inputs pertinent to our discussion are depicted.



With respect to this fundamental decentralized control question, we have been able to prove the following important new result:

Theorem: If $t_{ik}(s) = \dfrac{y_i(s)}{u_k(s)} = \dfrac{r_{ik}(s)}{p_{ik}(s)} \neq 0$, and $y_i(s)$ is fed back to $u_{j\neq k}(s)$ via g_{ji}, then all modes controllable via $u_k(s)$ before feedback will remain

<u>controllable via</u> $u_k(s)$ <u>irrespective of</u> g_{ji} (although certain of these u_k controllable modes could vary with g_{ji}). <u>In addition, all modes controllable via</u> u_j, <u>but not controllable via</u> u_k, <u>will become controllable via</u> u_k <u>for almost any</u> g_{ji} <u>except those which are both (i) unobservable at</u> y_i <u>and (ii) zeros of</u> $r_{ik}(s)$, <u>the prime</u> $\frac{y_i(s)}{u_k(s)}$ <u>transfer function</u>.

We now note that in view of Theorem 1 and the given (3 x 3) T(s), if $y_{3=i}$ is fed back to $u_{1=j}$ via g_{13}, then the $u_{2=k}$ controllable modes at $s = -4$ and $s = -5$ will remain controllable via u_2 as will the "new" u_2 controllable mode at $s = -3+g_{13}$. In addition, the u_1 controllable mode at $s = -1$ will become controllable via u_2 for almost any g_{13}. However, the u_1 controllable mode at $s = -2$ will <u>not</u> become controllable via u_2 after feedback because it is (i) unobservable at y_3 and (ii) a zero of $\frac{y_3(s)}{u_2(s)} = \frac{s+2}{(s+3)(s+5)}$. Finally, it might be noted that if we now set $k = 3$, our theorem clearly indicates that the u_3 controllable modes at $s = -1$ and -6 will remain u_3 controllable. In addition, the u_1 controllable modes at -2, and $-3 + g_{13}$ will become controllable via u_3 for almost any g_{13} after y_3 to u_1 feedback.

We merely remark here that a "dual" result involving y_k observable modes after y_i to u_j feedback via g_{ji} has also been formally stated and established. Furthermore, when combined, these two results provide a direct algorithm for determining the changes which occur in all (pxm) elements of the closed loop transfer matrix of the system. Such results can also be used to develop new control techniques for multivariable systems under decentralized control constraints. We are currently working on the development of such techniques.(48)

It has long been of interest to find explicit conditions for a real, canonical linear system $(C_{pxn},A_{nxn},B_{nxm})$ to be completely assignable: i.e. to have the property that for each real, monic, nth degree polynomial

$$\alpha(\lambda) = \lambda^n + \sum_{i=1}^{n} a_i\lambda^i \qquad \text{(A)}$$

there exists a real, constant output-feedback matrix F_{mxp} for which

$$\det(\lambda I-A-BFC) = \alpha(\lambda) \qquad \text{(B)}$$

It has also been of interest to identify those systems which are 'generically assignable': (C,A,B) is generically assignable if the set of all coefficient vectors $(a_1,a_2,\ldots,a_n)'$ associated with (A) for which thee exist real F satisfying (B) is open and dense in R^n.

Apart from the cases when either C has independent columns or B has independent rows (which can be dealt with using state-feedback theory), little is known about either complete or generic assignability. Perhaps the sharpest result to date, due to Kimura (49) and others, asserts that for 'almost every' linear system, generic pole assignment is possible provided $n \leq m + p - 1$. Willems and Hesselink (50) show that for almost every system with $m = p = 2$ and $n = 4$, generic pole assignment is not possible, even though the number of free parameters in F equals n. Using a vesion of the (51) implicit function theorem, Hermann and Martin prove that for almost every linear system whose first n Markov matrices, $CB,CAB,\ldots,{}^{n-1}B$ are linearly independent, generic pole assignment is possible provided F is allowed to be complex-valued. Brockett and Byrnes (52) take advantage of certain classical ideas based on elimination theory to devleop a formula which gives values of m,n and p, for which almost every linear system is generically assignable -but their development is not constructive.

In (53), we develop a useful formulation of the assignment problem for $p = 2$ and provide constructive solutions for some special cases. For $m = 2$ and $n = 4$ we characterize the classes of generically assignable and completely assignable systems. We show that for $m = 3$ and $n = 6$, almost every linear system is generically assignable; we do this by reducing the assignment problem to the problem of finding a real root of a real 5th degree polynomial in one variable. That such a polynomial should exist is in agreement with the Brockett-Byrnes formula derived in (52).

8. Digital Control of Multivariable Systems

Perhaps the most important new issue in multivariable control, is the question of actual implementation of the final design. Whereas ten years ago many of the designs were too complex and expensive to implement, microprocessor technology offers an exceptional opportunity for overcoming this limitation. Currently, there is a revolution in what sorts of controller designs can and should be physically employed in practice. Because of the advent of low cost microprocessors, future designs will be not only more reliable, but also less expensive. The proper and effective utilization of microprocessors for the control of complex multivariable processes is a new and exciting research area which presents many challenges to the control engineer.

It may be noted at this time that much as been done and is known regarding discrete control, expecially in the scalar case, i.e., texts and formal articles dealing with sampled data systems have been commonplace for years and are too numerous to mention. In most of these texts and articles however, microprocessors are not explicitly mentioned as the discrete time device and, perhaps more importantly, the limited memory, wordlength, and speed associated with microprocessors are not usually taken into consideration. Most of the current theory as well as the applications of digital control still assume a rather simple single input/output system to be controlled, usually by a discrete device with sufficient memory, wordlength, and speed to approximate that which would be attainable using a continuous controller. Nonetheless, microprocessor technology has given renewed impetus to the area of discrete control, and contemporary texts and articles in this field are becoming increasingly aware of the potential as well as the limitations of microprocessors.

With respect to microprocessor control of complex multivariable systems, much theoretical as well as practical work remains to be done. The work already accomplished in this area involves the first complete solution to deadbeat error tracking in discrete multivariable systems (54). In particular, by employing a polynomial matrix representation for the dynamical behavior of a discrete system in the delay operator d (which is Z-transformable to z^{-1}), the necessary and sufficient condition has been found for achieving a discrete error signal between reference input and plant output. The condition involves the skew primeness of two polynomial matrices in d, and extends the internal model principle to the discrete case. Work is proceeding in actually applying the theory to the microprocessor control of a linear motor in our new Laboratory for Engineering Man/Machine Systems (LEMS).

References

(1) P.L. Falb and W.A. Wolovich, "Decoupling in the design and synthesis of multivariable systems", IEEE AC-12, pp. 651-659, 1967.

(2) W.A. Wolovich and P.L. Falb, "On the structure of multivariable systems", SIAM J. on Control, Vol. 7 (4), August 1969.

(3) W.A. Wolovich and R. Shirley, "A frequency domain approach to handling qualities design", Proc. 1970 Joint Automatic Control Conference, Atlanta, 1970.

(4) P.L. Falb, Linear Systems and Variants, Lecture Notes, Control Group Lund University, Sweden, 1974.

(5) W.A. Wolovich and P.L. Falb, "Invariants and canonical forms under dynamic compensation", SIAM J. on Control, Vol. 14 (6), November 1976.

*(6) P.L. Falb and W.A. Wolovich, "Invariants and canonical forms under feedback", Lefschetz Center for Dynamical Sytstems Report, LCDS 78-2, 1978.

*(7) P.L. Falb and W.A. Wolovich, "The role of the interactor in decoupling", Joint Automatic Control Conference, San Francisco, 1977.

*(8) H. Elliott, Adaptive Control and Identification of Linear Systems, Ph.D. Dissertation, Brown University, January 1978.

*(9) H. Elliott and W.A. Wolovich, "Parameter adaptive identification and controll", Brown University Technical Report, ENG FE78-1, 1978.

*(10) V. Eldem, Parameter Insensitive Pole Placement in Linear Systems, Ph.D. Dissertation, Brown University, 1978.

*(11) H. Elliott and W.A. Wolovich, "A frequency domain model reduction procedure", Automatica, Vol. 16, pp. 168-178, Feb. 1981.

*(12) W.A. Wolovich, "Multipurpose controllers for multivariable systems", IEEE Trans. Auto. Control, Vol. AC-26 (1), pp. 162-170, Feb. 1981.

*(13) C. Byrnes and P.L. Falb, "Applications of algebraic geometry in system theory", Amer. J. of Math., Vol. 101, pp. 337-363, 1979.

(14) I. Postlethwaite, J.M. Edwards. A.G.J. MacFarlane, "Principal gains and principal phases in the analysis of linear multivariable feedback systems", IEEE Trans. Auto. Control, Vol. AC-26 (1), pp. 32-46, Feb. 1981.

(15) A.G.J. MacFarlene and I. Postlethwaite, "The generalized Nyquist stability criterion and multivariable root loci", Int. J. Control, Vol. 25, pp. 81-127, 1977.

(16) I. Postlenthwaite and A.G.J. MacFarlane, A Complex Variable Approach to the Analysis of Linear Multivariable Feedback Systems, Berlin Springer, 1979.

(17) H.H. Rosenbrock, Computer Aided Control System Design, Academic Press, 1974.

(18) C. Byrnes, "Algebraic and geometric aspects of the analysis of feedback systems", in Geometric Methods for the Theory of Linear Systems, D. Reidel, Dordrecht, 1980.

(19) C. Byrnes, T. Duncan, "Toipological and geometric invariants arising in control theory", to appear.

(20) N.A. Lohtomaki, N.R. Sandell, Jr., M. Athans, "Robustness results in linear-quadratic gaussian based multivariable control designs", IEEE Trans. Auto. Control, Vol. AC-26 (1), pp. 75-93, Feb. 1981.

(21) M.G. Safanov, Stability and Robustness of Multivariable Feedback Systems, MIT Press, 1980.

(22) E.J. Davidson, I.J. Ferguson, "The design of controllers for the multivariable robust servomechanims problem using parameter optimization methods", IEEE Trans. Auto. Control, Vol. AC-26 (1), pp. 93-110, Feb. 1981.

(23) L. Cheng, J.B. Pearson, "Synthesis of linear multivariable regulators", IEEE Trans. Auto. Control, Vol. AC-26 (1), pp. 194-202, Feb. 1981.

(24) A.S. Morse, "Global stability of parameter adaptive control systems", IEEE Trans. Auto. Control, Vol. AC-25 (3), pp 433-439, June 1980.

(25) K.S. Narendra, Y-H. Lin, L.S. Valavani, "Stable adaptive controller design, part II: proof of stability", IEEE Trans. Auto. Control, Vol. AC-25 (3), pp. 440-448, June 1980.

(26) G.C. Goodwin, P.J. Ramadge, P.E. Caines, "Discrete time multivariable adaptive control", IEEE Trans. Auto. Control, Vol. AC-25 (3), pp. 449-456, June 1980.

(27) W.A. Wolovich, Linear Multivariable Systems, Springer-Verlag, 1974.

*(28) H. Elliott, W.A. Wolovich, "A parameter adaptive control structure for linear multivariable systems", IEEE Trans. Auto. Control, Vol AC-27 (3), April 1982.

(29) H. Elliott, "Hybrid adaptive control of continuous time systems", Colorado State Tech. Report, #JL80-DELENG-1, July 1980 (to appear IEEE Trans. Auto. Control).

(30) H. Elliott, "Direct adaptive pole placement with applications to nonminimum phase systems", IEEE Trans. Auto. Control Vol. AC-26, June 1982.

*(31) H. Elliott and W.A. Wolovich, "Multivariable adaptive pole-placement", Proceedings - 20th Conference on Decision and Control, San Diego, Dec. 1981.

*(32) H. Elliott, W.A. Wolovich, and M. Das, "Arbitrary adaptive pole placement for linear multivariable systems", Colorado State University Tech. Report. No. #FB82-DELENG-2, February 1982.

(33) M.J. Bosley and F.P. Les, "A survey of simple transfer-fyunction derivations from high-order state variable models", Automatica, 8, 765-776 (1972).

(34) E.J. Davidson, "A method for simplifying linear dynamic systems", IEEE Trans. AC-11, 93-101 (1966).

(35) C.F. Chen and L.S. Sheh, "A novel approach to linear model simplification", Int. J. Control 8, 561-570 (1968).

(36) M. Hutton and B. Friedland, "Route approximation for reducing order of linear time-invariant systems", IEEE Trans. AC-20, 329-337 (1975).

(38) P.A. Payne, "An improed technique for transfer function synthesis from frequency response data", IEEE Trans. AC-15, 480-483 (1970).

(39) N.K. Sinha and W. Pille, "A new method for reduction of dynamic systems", Int. J. Control, 14, 111-118 (1971).

(40) P.M. Lion, "Rapid identification of linear and nonlinear systems", AIAA Journal, 5, 1835-1842 (1967).

*(41) W.A. Wolovich, "On the stabilization of closed loop stabilizable systems" IEEE Trans. Auto. Control, Vol. AC-24 (3) June 1979.

*(42) W. A. Wolovich, "Skew prime polynomial matrices" IEEE Trans. Auto. Control Vol. AC-23 (5), October 1978.

*(43) W.A. Wolovich and P. Ferreira, "Output regulation and tracking in linear multivariable systems", IEEE Trans Auto. Control Vol AC-24 (3) June 1979.

(44) E.J. Davidson, "The robust control of a servomechanims problem for linear time-invariant multivariable systems", IEEE Trans. Auto. Control Vol. AC-21 Feb. 1976, pp. 25-34.

(45) C.A. Doesoer and Y.T. Want, "On the minimum order of a robust servo-compensator" IEEE Trans. Auto. Control Vol. AC-23, Feb. 1978, pp. 70-73.

(46) W.A. Wolovich, "Output feedback decoupling" IEEE Trans. Auto Cont. Vol. AC-20 (1), Feb. 1975, pp. 148-149.

*(47) W.A. Wolovich, "Scalar output feedback in linear multivariable systems" Proceedings VIII IFAC Contress, KYOTO, JAPAN, August 24-28, 1981.

*(48) T.J. Huang, Constrained Control of Linear Multivariable Systems, Ph.D. Dissertation, Division of Engineering, Brown University, May 1982.

(49) H. Kimura, "Pole assignment by gain output feedback", IEEE Trans. auto Control, AC-20, pp. 509-516.

(50) J.C. Williams and W.H. Hesselink, "Generic properties of the pole placement problem", Proc. 1978 IFAC, Helsinki.

(51) R. Hermann and C.F. Martin, "Applications of algebraic geometry to systems theory - Part I", IEEE Trans. Auto. Contr., AC-22, pp. 19-25.

(52) R.W. Brockett and C.I. Byrnes, "Multivariable Nyquist criterion, root loci over pole placement: a geometric viewpoint", IEEE Trans. Auto. Control AC-26 (1), Feb. 1981, pp. 271-283.

*(53) A.S. Morse, W.A. Wolovich, and B.D.O. Anderson, "Generic pole assignment: preliminary results", Proceedings 1981 IEEE Conference on decision and control, San Diego, CA, December 1981.

*(54) W.A. Wolovich, "Deadbeat error control of discrete multivariable systems", Brown University Technical Report #ENG SE81, September 1981. Also to appear in the IEEE Trans. Auto. Control.

*Denotes research done under AFOSR sponsorship during the grant period.

DATE

FILMED

8-8